

Longwave Interface Instability In Two-Fluid Vibrational Flow ¹

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Abstract

We consider longwave mode of the interface instability in the system comprising of two immiscible fluid layers. The fluids fill out plane horizontal cavity which is subjected to horizontal harmonic vibration. The analysis is performed within the framework of "high frequency of the vibration" approximation and the averaging procedure. The nonlinear equation (having the form of Newton's second law) for the amplitude of interface deformation is obtained by means of multiple scales method. It is shown that (in addition to previously detected quasistationary periodic solutions) the equation has a class of quasistationary solitary solutions.

In experimental works by Bezdeneznykh *et al.* [1] and by Wolf [5] for a long horizontal reservoir filled with two immiscible viscous fluids, an interesting phenomenon was found at the interface: the horizontal vibrations lead to the formation of a steady relief. This formation mechanism has a threshold nature; it is noteworthy that such a wavy relief appears on the interface only if the densities of the two fluids are close enough, i.e. it does not appear for the liquid/gas interface (free surface). The interface is absolutely unstable if the heaviest fluid occupies the upper layer; i.e., the horizontal vibration does not prevent the evolution of Rayleigh-Taylor instability, in contrast to the vertical one which under certain conditions suppresses its evolution. A theoretical description of this phenomenon was provided by Lyubimov & Cherepanov [3] within the framework of a high frequency (of the vibration) approximation and an averaging procedure; they found that a horizontal vibration leads to a quasistationary state i.e., a state where the mean motion is absent but the interface oscillates with a small amplitude (of the order of magnitude of the cavity displacement) with respect to the steady relief. They also obtained the general equations and boundary conditions for

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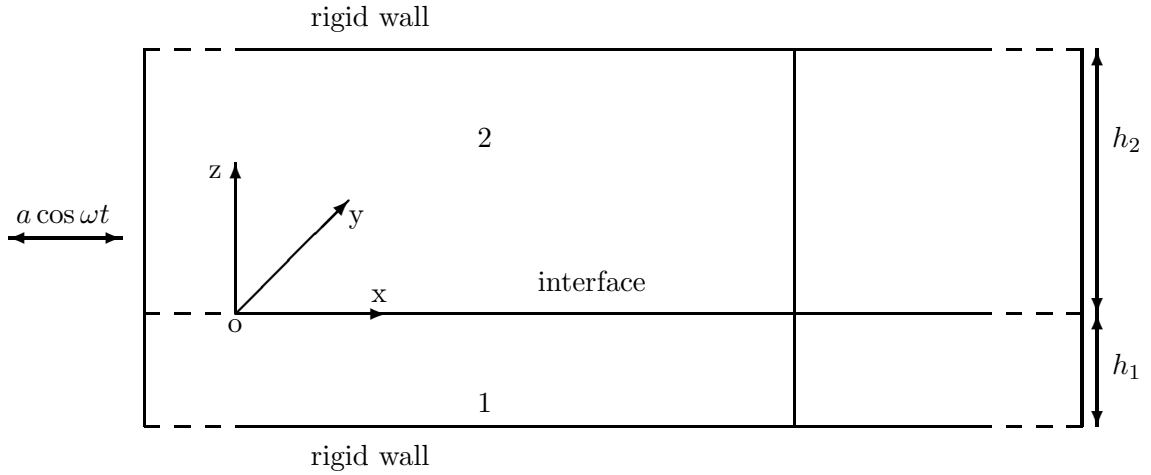


Fig. 1. Problem configuration.

mean and pulsational parts of the fluid velocities. The theory developed in [3] made it possible to perform the linear stability analysis for the interface.

In the approach [3], two parameters were assumed to be asymptotically small simultaneously: (i) the dimensionless thickness of the viscous skin-layers $\delta = h^{-1}\sqrt{\nu/\omega}$, ν being the kinematic viscosity and (ii) the dimensionless amplitude of the vibration $\epsilon = a/h$. In this limiting case, the possibility of description of parametric resonant effects is absent and only the basic instability mode (Kelvin–Helmholtz, of two counter flows) remains.

The linear stability analysis for inviscid and viscous fluids in the case of finite ϵ and relatively low frequencies of the vibration was carried out analytically and numerically in [2], [4]. The transformation was found which reduces the linear stability problem under inviscid approximation to the Mathieu equation. The parametric resonant regions of instability associated with the intensification of capillary-gravity waves at the interface and the effects due to viscous damping were examined.

In the present work, following the approach of [3], we make the analytical study for longwave interface instability in the high-frequency vibrational field.

1. Let us consider the system of two immiscible, incompressible liquids filling rectangular cavity of length L and height h . In the state of rest the heavy liquid (of density ρ_1) occupies the bottom region of height h_1 , and the light liquid (of density ρ_2) – the upper region of height h_2 ($h = h_1 + h_2$). We choose Cartesian coordinate system in such a way that the x, y -axis lie in horizontal plane, the z -axis is directed vertically, $z = 0$ corresponds to the unperturbed interface (Fig. 1). Let the cavity perform harmonic oscillation along the x -axis, with the amplitude a and frequency ω .

In the basic state (which is a counter flow), the interface could be considered as plane and horizontal. For discussion of this issue as well as the approximation of infinite horizontal layer we

refer the readers to [2].

The quasistationary perturbed state is found from the following problem:

$$\Delta\Psi = 0, \quad \Delta\Phi = 0, \quad (1)$$

$$z = -H_1 : \quad \Psi = 0; \quad z = H_2 : \quad \Phi = 0, \quad (2)$$

$$z = \xi(x) :$$

$$\Psi - \Phi = \frac{(\rho - 1)(H_1 + H_2)}{\rho H_2 + H_1} \xi, \quad (3)$$

$$\rho(\Psi_z - \Psi_x \xi_x) = \Phi_z - \Phi_x \xi_x, \quad (4)$$

$$B \left[\frac{\rho(H_1 + H_2)}{\rho H_2 + H_1} \Psi_z + \frac{H_1 + H_2}{\rho H_2 + H_1} \Phi_z + \Psi_z \Phi_z + \Psi_x \Phi_x \right] - \xi + \frac{\xi_{xx}}{(1 + \xi_x^2)^{3/2}} = const, \quad (5)$$

$$\rho = \rho_1/\rho_2, \quad B = \frac{a^2 \omega^2}{4} \left(\frac{\rho_1 - \rho_2}{\alpha g} \right)^{1/2}, \quad H_1 = h_1/L, \quad H_2 = h_2/L,$$

where Ψ and Φ are the streamfunctions of small 2D normal perturbations, α is the coefficient of the interface tension, ξ is interface deformation. Equations (1)-(5) are in dimensionless form; the length scale is $L = [\alpha/(\rho_1 - \rho_2)g]^{1/2}$ and this is also the scale for Ψ and Φ . The x, z differentiation is denoted by the respective subscripts. In the case of equal heights of the layers ($H_1 = H_2 = H$), the equations (1)-(5) are reduced to equations (2.6)-(2.7) in [3].

2. Let us consider the amplitude $\xi(x)$ of interface deformation to be small, $\xi \ll 1$. This allows to impose boundary conditions on the unperturbed interface, $\xi = 0$, being accurate up to linear terms. According to the main idea of multiple scales method, we introduce the set of lengths,

$$x_1 = \epsilon x, \quad x_2 = \epsilon^2 x, \quad x_3 = \epsilon^3 x, \dots$$

and we assume that all variables in (1)-(5) are the functions of these lengths. Then we have the following expansions:

$$\frac{\partial f}{\partial x} = \epsilon \frac{\partial f}{\partial x_1} + \epsilon^2 \frac{\partial f}{\partial x_2} + \epsilon^3 \frac{\partial f}{\partial x_3} + \dots, \quad \Psi = \Psi_2 \epsilon^2 + \Psi_4 \epsilon^4 + \dots, \quad (6)$$

$$\Phi = \Phi_2\epsilon^2 + \Phi_4\epsilon^4 + \dots, \quad \xi = \xi_2\epsilon^2 + \xi_4\epsilon^4 + \dots$$

The parameter B , characterizing the vibration intensity, is represented like $B = B_* + \epsilon^2 r$, B_* being the threshold instability value with respect to longwave perturbations, r is super(under)criticality parameter.

From (1)-(5) we get the following relations in the leading order of the expansion in ϵ :

$$\Psi_2 = \frac{(\rho - 1)(H_1 + H_2)H_1}{(\rho H_2 + H_1)^2} \xi_2 \left(1 + \frac{z}{H_1}\right), \quad \Phi_2 = \frac{\rho(1 - \rho)(H_1 + H_2)H_2}{(\rho H_2 + H_1)^2} \xi_2 \left(1 - \frac{z}{H_2}\right), \quad (7)$$

$$B_* = \frac{(\rho H_2 + H_1)^3}{2\rho(\rho - 1)(H_1 + H_2)^2}. \quad (8)$$

The equation (8), in case of equal heights of the layers, is reduced to

$$B_* = \frac{H(\rho + 1)^3}{8\rho(\rho - 1)},$$

which is the correct instability threshold value [3].

In the next order we have (instead of ξ_2 we write just ξ):

$$\Psi_4 = C_1 z^3 + C_2 z^2 + C_3 z + C_4, \quad \Phi_4 = C_5 z^3 + C_6 z^2 + C_7 z + C_8, \quad (9)$$

$$\xi_{xx} \left(1 - \frac{H_1^3 + \rho H_2^3}{3(\rho H_2 + H_1)}\right) + \frac{2\rho(\rho - 1)(H_1 + H_2)^2}{(\rho H_2 + H_1)^3} r \xi + \frac{3(\rho - 1)}{\rho H_2 + H_1} \xi^2 = C, \quad C = \text{const}. \quad (10)$$

Here $C_1 - C_8$ are known functions of $\rho, H_1, H_2, \xi(x)$.

It was shown in [3] that in thin ($H^2 < 3$) layers of equal thickness H the most dangerous are longwave perturbations (in the sense that they appear at the smallest possible destabilization amplitude). The same longwave perturbations in the layers of different heights are the most dangerous if $(H_1^3 + \rho H_2^3)/(\rho H_2 + H_1) < 3$. In the following, we consider this case only. Besides, we are interested only in the solutions to (10) which are zero at infinity, i.e. for such solutions $C = 0$.

The nonlinear equation (10) could be rewritten in the form of Newton's 2nd law:

$$\xi_{xx} = -dU/d\xi, \quad U = \frac{1}{2} q r \xi^2 + \frac{1}{3} p \xi^3, \quad q(\rho, H_1, H_2) > 0, \quad p(\rho, H_1, H_2) > 0. \quad (11)$$

The function $U(\xi)$ (potential energy) is presented in Fig. 2, for different values $r < 0$ ($r = -0.4, -0.6, -0.75$).

To zero level of U corresponds the deformation of the interface which has the form of quasi-stationary soliton (Fig. 3). It's amplitude, as follows from (11), is given by

$$\xi_m = \frac{3}{2} \frac{q}{p} |r|.$$

The soliton is stable, since the values $r < 0$ correspond to undercriticality. This gives hope to observe it in the future experiments.

It is noteworthy that besides the solutions of solitary type to eq. (11), in the space of parameters we find also periodic solutions, which correspond to values $U < 0$. These solutions were examined in details in [3], [6].

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